

END TERM EXAMINATION

SECOND SEMESTER [B.TECH] JUNE 2025

Paper Code: BS-112

Subject: Applied Mathematics-II

Time: 3 Hours

Maximum Marks:60

Note: Attempt five questions in all including Q.No.1 which is compulsory. Select one question from each unit.

- Q1 Attempt all questions (5x4=20)
- Prove that $f(z) = \bar{z}$ is nowhere analytic.
 - Find the Principal value of $\log(-1 - i)$
 - Find the Laplace transformation of $(1 + t)e^{3t}$.
 - Taylor series expansion of $\frac{1}{z-2}$ in $|z| < 1$ is

UNIT-I

- Q2
- Find all the values of $(1 + i)^i$. (5)
 - Verify that the function $u(x, y) = x(1 + y)$ is harmonic. Find its corresponding analytic function. (5)
- Q3
- Evaluate the integral $\oint_C \tan z \, dz$ along the simple closed curve defined by $C : |z| = 5$. (5)
 - Find the Line integral $\int_{\frac{1}{2}}$ along the straight line with the end points $z = 0$ and $z + 1 + i$. (5)

UNIT-II

- Q4
- Find the Bilinear transformation or Mobius transformation which maps $1, -i, -1$ of the z -plane onto the points $i, -1, i$ of the w -plane respectively. (5)
 - Discuss the type of singularities and find their residues for the function $f(z) = \frac{1}{\sin z}$. (5)
- Q5
- Find and plot the Image of the vertical line $x = 1$ as well as horizontal line $y = 0$ under the function $f(z) = e^z$. (5)
 - Prove that the integral $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx = \frac{\pi}{3}$. (5)

UNIT-III

- Q6
- Using Laplace transformation solve the ordinary differential equation $y''(t) - 3y'(t) + 2y(t) = e^{3t}$, $y(0) = 1, y'(0) = 0$. (5)
 - Find the inverse Laplace transformation of $F(s) = \frac{-11s+25}{s^2-6s+15}$. (5)
- Q7
- Determine the Fourier series of the function $f(x) = 1 - x^2$ in the interval $[-1, 1]$. (5)
 - Find the Fourier transformation of $f(x) = \begin{cases} -1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$
Hence evaluate $\int_0^{\infty} \frac{\sin x}{x}$. (5)

UNIT-IV

- Q8 a) Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature zero, assuming that initial temperature is $f(x) = \begin{cases} x, & 0 < x < \frac{L}{2} \\ L - x, & \frac{L}{2} < x < L \end{cases}$ (5)
- b) A tightly stretched string with fixed end points $x = 0$ and $x = l$, is initially in a position given by $y = \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from its position, find the displacement $y(x, t)$. (5)
- Q9 a) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, subject to the conditions $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin \frac{n\pi x}{l}$. (5)
- b) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, with boundary conditions $u(x, 0) = 3 \sin(n\pi x)$, $u(0, t) = 0$ and $u(1, t) = 0$, where $0 < x < 1, t > 0$. (5)
