(Please write your Exam Roll No.) Exam Roll No. ....

END TERM EXAMINATION

SECOND SEMESTER [B.TECH] JUNE 2024

Paper Code: BS-112 **Subject: Applied Mathematics-II**

Time: 3 Hours **Maximum Marks: 60**

Note: Attempt any five questions including Q.No1 which is compulsory. Select one question from each unit.

Q1 Attempt all of the following:- **(3x4=12)**

(a) Resolve $e^{\sin(x+iy)}$ into real and imaginary parts

(b) Find the image of infinite strip $\frac{1}{4} \le y \le \frac{1}{2}$. under the transformation $w = \frac{1}{z}$.

(c) Find the Laplace transform of $\frac{Cosat - Cosbt}{t}$.

(d) Using the method of Separation of variable, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$

Where $(x, 0) = 6e^{-3x}$.

UNIT-I

Q2 (a) Find the value of C_1 and C_2 such that the function **(6)**

$f(z) = x^2 + C_1y^2 - 2xy + i(C_2x^2 - y^2 + 2xy)$ is analytics. Also find $f'(z)$.

(b) Show that the function $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic. Find its harmonic Conjugate. **(6)**

Q3 (a) State Cauchy Integral formula and hence evaluate **(6)**

$\int_C \frac{3z^2 + z}{z^2 - 1} dz$, where C is the circle $|z - 1| = 1$.

(b) Evaluate the Line Integral $\int_C z^2 dz$, Where C is the boundary of a triangle with vertices $0, 0 + i, -1 + i$, Clockwise. **(6)**

UNIT-II

Q4 (a) Evaluate $\int_C \frac{z^2}{(z-1)^2(z+2)} dz$, Where C is $|z| = 3$ by using Cauchy residue theorem. **(6)**

(b) Evaluate Laurents series which represents the function **(6)**

$f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ When (i) $|z| < 2$, (ii) $2 < |z| < 3$.

Q5 (a) Apply the calculus of residues to evaluate **(6)**

$\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+a^2)(x^2+b^2)} dx$, $a > b > 0$.

(b) Let $f(z)$ be a bilinear transformation such that $f(\infty) = 1$, $f(i) = i$, and $f(-i) = -i$

Find the image of unit dick (c) $\{z \in c; |z| < 1\}$ under (z). **(6)**

P.T.O.

UNIT-III

Q6 (a) Find the Inverse Laplace transform of $F(s) = \log\left[\frac{s+a}{s+b}\right]$. **(6)**

(b) Find the Fourier series for $f(x) = -\pi$, $-\pi < x < 0$

$= x$, $0 < x < \pi$

And deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots\ldots\ldots\ldots = \frac{\pi^2}{8}$. **(6)**

Q7 (a) Solve $\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = 0$, Given $y = \frac{dy}{dt} = 0, \frac{d^2y}{dt^2} = 6$ at $t = 0$. **(6)**

By using Laplace transformation .

(b) Find Fourier Transform of $f(x) = \begin{cases} 1, |x| < 1 \\ 0, \ |x| > 1 \end{cases}$ **(6)**

Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$.

UNIT-IV

Q8 (a) A String is stretched and fastened to two points l apart. Motion is started By displacing the string in the form $y = a\sin\frac{\pi x}{l}$ from which it is released at Time $t = 0$. Show that the displacement of any point at a distance $x$from One end at time t is given by $y(x,t) = a\sin(\frac{\pi x}{l})\cos(\frac{\pi ct}{l})$. **(6)**

(b) Find the temperature in a bar of length 2 whose ends are kept at zero and lateral Surface insulated if the initial temperature is $\sin\frac{\pi x}{2} + 3\sin\frac{5\pi x}{2}$.**(6)**

Q9 (a) An infinitely long plane uniform plate is bounded by two parallel edges and an end At right angles to them. The breadth is π. This end is maintained at temperature u_0 At all points and the other edges are at zero temperature. Determine the temperature At any point of the plate in the steady state.**(6)**

(b) Solve $\frac{1}{4}u_{xx} = u_{tt}$, With Initial Condition **(6)**

$u(x,0) = 0$, $u_t(x,0) = 8\sin 2x$. using by D'Alembert Principal.

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