# END TERM EXAMINATION

**FOURTH SEMESTER [B.TECH] JULY 2023**

| **Paper Code: BS-202** | **Subject: Probability, Statistics and Linear Programming** |
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| **Time: 3 Hours** | **Maximum Marks: 75** |

***Note: Attempt five questions in all including Q.No.1 which is compulsory. Select one question from each unit.***

Q1 Attempt all questions:

a) Three urns contain 6 red, 4 black; 4 red, 6 black, and 5 red, 5 black balls respectively. One of the urns is selected and a ball is drawn from it. If the ball drawn is red, find the probability that it is drawn from the first urn. **(3)**

b) A manufacturer produces two types of models $M_1$ and $M_2$. Each $M_1$ model requires 4 hours of grinding and 2 hours of polishing; whereas $M_2$ model requires 2 hours of grinding and 5 hours of polishing. The manufacturer has 2 grinders and 3 polishers. Each grinder works for 40 hours a week and each polisher works for 60 hours a week. Profit on an $M_1$ model is ₹ 3 and on an $M_2$ model is ₹ 4. Whatever is produced in a week is sold in the market. How should the manufacturer allocate his production capacity to the two types of models so that he may make the maximum profit in a week. Formulate this as a linear programming problem. **(3)**

c) Determine the value of c that makes the function $f(x, y) = c(x + y)$, a joint probability mass function over points $x = 1, 2, 3$ and $y = 1, 2, 3$. Find E(X) and E(Y). **(3)**

d) Find the line of regression or line of best fit for the data given below: $n = 18,\ \sum x = 12, \sum y = 18, \sum x^2 = 60, \sum y^2 = 96, \sum xy = 48$. **(3)**

e) Find the moment generating function for the distribution defined by $f(x) = \frac{1}{2^x},\ x = 1, 2, 3, \ldots \infty$ **(3)**

## UNIT-I

Q2 a) Show that Poisson distribution is a limiting case of Binomial distribution when n is very large and p is small such that np is fixed. If $X$ is a Poisson variate such that $P(X = 2) = 9\,P(X = 4) + 90\,P(X = 6)$. Find the mean and standard deviation of $X$. **(8)**

b) In the normal distribution, 31% of the items are under 45 and 8% are over 64. Determine the mean and variance of the distribution? (Given that $P(Z < -1.4) = 0.08$ and $P(Z > 0.5) = 0.31$). **(7)**

Q3 a) There is an 85% chance that a student will pass a Statistics course. A random sample of 120 students taking the Statistics course is selected. Using Normal approximation (using continuity correction), find the probability that

(i) atleast 110 students will pass.

(ii) atmost 105 students will pass.

(Given that $P(Z > 1.92) = 0.0274$ and $P(Z > 0.89) = 0.1867$). (8)

b) If $X$ follows uniform distribution in $[-2, 2]$, find $P(X < 0)$ and $P\left(|X - 1| < \frac{1}{2}\right)$. (7)

## UNIT-II

Q4 a) Find covariance and correlation for joint probability density function of the two random variables $X$ and $Y$:

$f(x, y) = e^{-x-y}$ over range $0 < x, 0 < y$.

Are $X$ and $Y$ independent random variables? (8)

b) A random sample of size $n_1 = 16$ is selected from a normal population with mean 75 and standard deviation 8. A second random sample of size $n_2 = 9$ is taken from another normal population with mean 70 and standard deviation 12. Let $\overline{X_1}$ and $\overline{X_2}$ be the sample mean of the first and second sample respectively. Find the probability that $\overline{X_1} - \overline{X_2}$ exceeds 4. (Given $P(Z > -0.2236) = 0.5871$). (7)

Q5 a) Suppose that a random variable $X$ has a continuous uniform distribution

$$f(x) = \begin{cases} \frac{1}{2}, & 4 \le x \le 6 \\ 0, & \text{otherwise.} \end{cases}$$

(i) State Central Limit Theorem and use it to find the approximate probability distribution of the sample mean of a random sample of size $n = 40$.

(ii) Find the mean and variance of the sample mean. (10)

b) You are given a bag of marbles. Inside the bag are 5 red, 4 white and 3 blue marbles. Using the concept of multinomial probability distribution, calculate the probability that with 6 trials, you choose 3 marbles that are red, 1 marble that is white, and 2 marbles that are blue, replacing each marble after it is chosen. (5)

## UNIT-III

Q6 a) In one sample of 8 observations, the sum of squared of deviations of the sample values from the sample mean was 84.4 and in the other sample of 10observations it was 102.6. Test whether the difference is significant at5% level of significance? (Given $F_{0.05}[7, 9] = 3.29$). (8)

b) Following are the responses to the question "How many hours do you study before a major Statistics test?"

6 5 1 2 2 5 7 5 3 7 4 7

Use the Wilcoxon test to test the hypothesis at the 5% level of significance that the median number of hours a student studies before a test is 3. (Given that, the critical value of T for $n = 11$ at 5% level of significance for a two-tailed test is 10). [7]

Q7 a) The theory predicts the proportion of beans in the four groups should be in the ratio 9:3:3:1. In an experiment with 1600 beans the numbers in the four groups were 882, 313, 287 and 118. Does the experimental result support theory? (Given $\chi^2_{0.05}$ for 3 *d. f.* = 7.815). [8]

b) The means of samples of sizes 1000 and 2000 are 67.5 and 68.0 cm respectively. Can the samples be regarded as drawn from the same population of standard deviation 2.5 cm. (Given $Z_{0.05}$ = 1.96). [7]

## UNIT-IV

Q8 a) Solve the following L.P.P by Simplex method:

$$\text{Maximize } Z = 5x_1 + 3x_2$$

$$\text{subject to } x_1 + x_2 \le 2, 5x_1 + 2x_2 \le 10, 3x_1 + 8x_2 \le 12, x_1, x_2 \ge 0.$$ [8]

b) Construct the dual of the following L.P.P and solve the dual using graphical method:

$$\text{Minimize } W = 2y_1 + 4y_2 + 3y_3$$

$$\text{subject to } -y_1 + y_2 + y_3 \ge 2, 2y_1 + y_2 \ge 1, y_1, y_2, y_3 \ge 0.$$ [7]

Q9 a) Using VAM method, find basic feasible solution of the following transportation problem. Check optimality and hence find the optimal solution. [8]

| | PLANT | | | | |
|---|---|---|---|---|---|
| **MARKET** | **P1** | **P2** | **P3** | **P4** | **Requirement** |
| **M1** | 19 | 14 | 23 | 11 | 11 |
| **M2** | 15 | 16 | 12 | 21 | 13 |
| **M3** | 30 | 25 | 16 | 39 | 19 |
| **Availability** | 6 | 10 | 12 | 15 | 43 |

b) Three jobs A, B, and C are to be assigned to three machines U, V, and W. The processing cost for each job machine combination is shown in the matrix given below. Determine the allocation that minimizes the overall processing cost. (Cost is in Rs. per unit). (7)

| JOB | MACHINE | | |
|---|---|---|---|
| | U | V | W |
| A | 17 | 25 | 31 |
| B | 10 | 25 | 16 |
| C | 12 | 14 | 11 |

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