

END TERM EXAMINATION

FIRST SEMESTER [B.TECH] DECEMBER 2024

Paper Code: BS-111

Subject: Applied Mathematics-I

Time: 3 Hours

Maximum Marks: 60

Note: Attempt any five questions including Q.No1 which is compulsory.
Internal Choice is indicated.

Q1. Attempt any Four of the following questions:

(4x5=20)

a) For the transformation $x = a(u + v), y = b(u - v)$ and

$$u = r^2 \cos 2\theta, v = r^2 \sin 2\theta, \text{ Find } \frac{\partial(x, y)}{\partial(r, \theta)}$$

b) Test for consistency the following equation and solve them if consistent
 $3x + 7y + 8z = 13, 2x + 9z = 5, -4x + y + 26z = 2$

c) If vector $\vec{v} = (x + 3y)\hat{i} + (y - 2z)\hat{j} + (x + az)\hat{k}$ is solenoidal vector, then find the value of a.

d) Find the directional derivative of $f(x, y, z) = xy^2 + yz^2$ at the point $(2, -1, 1)$ in the direction of vector $\hat{i} + 2\hat{j} + 3\hat{k}$

e) If $z = f(x + ay) + \phi(x - ay)$, then show that $z_{yy} = a^2 z_{xx}$.

f) Solve $\frac{dy}{dx} = \frac{x(x^2 + y^2 - 1)}{y(x^2 + y^2 + 1)}$

g) Define Gamma Function and Prove that $\int_0^{\infty} e^{-x^2} dx = \frac{1}{2}\sqrt{\pi}$

h) Verify the vectors $(1, 9, 9, 8), (2, 0, 0, 3), (2, 0, 0, 8)$ are linearly Independent or not?

Q2

a) If $\frac{x^2}{a^2 + u} + \frac{y^2}{b^2 + u} + \frac{z^2}{c^2 + u} = 1$, show that $u_x^2 + u_y^2 + u_z^2 = 2(xu_x + yu_y + zu_z)$. (5)

b) Show that the function $f(x, y) = x^3 \exp(-x^2 - y^2)$ has a maximum at the point $(\sqrt{3/2}, 0)$. A minimum at $(-\sqrt{3/2}, 0)$ and a stationary point at the origin whose nature cannot be determined. (5)

OR

Q3 a) Evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$ by applying differentiation under integral sign (5)

b) Find the stationary points of $f(x, y, z) = x^3 + y^3 + z^3$ subject to constraint $g(x, y, z) = x^2 + y^2 + z^2 = 1$ (5)

Q4 a) Solve $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$ (5)

b) Solve $(x^2y - 2xy^2)dx = (x^3 - 3x^2y)dy$ (5)

OR

Q5 a) Solve $(D^2 + 2D + 1)y = 2x + x^2$ (5)

~~b) Apply the method of variation of parameters, to solve $\frac{d^2y}{dx^2} + y = \text{Cosec } x$~~ (5)

Q6 a) Solve the following system by Gauss Elimination method (5)

$$2x_1 - x_2 + 3x_3 = 9$$

$$x_1 + x_2 + x_3 = 6$$

$$2x_1 - x_2 + x_3 = 2$$

b) If $\begin{vmatrix} a & a^2 & a^3 - 1 \\ b & b^2 & b^3 - 1 \\ c & c^2 & c^3 - 1 \end{vmatrix} = 0$ in which a, b, c are different, show that $abc = 1$. (5)

OR

Q7 a) State Cayley-Hamilton Theorem and verify Cayley-Hamilton Theorem for (5)

the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

b) Find the diagonalization for the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ (5)

Q8 a) Using Green's theorem find the line integral $\int_C (x^2 - 2xy)dx + (x^2y + 3)dy$ (5)

where C is the boundary of the region defined by $y^2 = 8x$ and $x = 2$

b) Find the directional derivative of $f(x, y, z) = xy^3 + yz^2$ at the point (2, -1, 1) in the direction of vector $\hat{i} + 2\hat{j} + 3\hat{k}$ <https://www.ggsipuonline.com> (5)

OR

Q9. a) Find the divergence and curl of $F = 2x^2z\hat{i} - xy^2z\hat{j} + 3yz^2\hat{k}$ at the point (1, 1, 1) (5)

b) Find the curvature and torsion for the curve $x = a \sin t, y = a \cos t, z = at \cot \alpha$ (5)
