(Please write your Exam Roll No.)

END TERM EXAMINATION

FIRST SEMESTER [B.TECH] DECEMBER 2025

Subject: Applied Mathematics-I

Paper Code: BS-111 | Maximum Marks: 60

Time: 3 Hours

Note: Attempt all questions as directed. Internal choice is indicated.

Q1 Attempt **any four** of the following questions: (4x5=20)

(a) If $z = x^y + y^x$. then Prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial x \partial y}$.

(b) Find the value of $\frac{d}{dx}\int_{x^2}^{x^3} \frac{1}{\log t} dt$.

(c) Solve the differential $(1+y^2)dx = (\tan^{-1} y - x)\, dy$.

(d) Express $4x^3 + 6x^2 + 7x + 2$ in term of Legendre's polynomials.

(e) Convert given Metrix A into normal form and find its rank:

$$\text{If } A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$$

$a^x = a^x \log a$

(f) Examine the following vectors for linear dependent and find the relation if it exists

$X_1 = (1,2,4)$, $X_2 = (2,-1,3)$, $X_3 = (0,1,2)$, $X_4 = (-3,7,2)$

(g) Determine the constant a, b, c so that $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational.

$\underset{c}{-1},\ \underset{a}{4},\ \underset{b}{2}$

(h) Show that $iv(grad r^n) = n(n+1)r^{n-2}$.

Q2 (a) If $W = f(x,y)$, $x = r\cos\theta$, $y = r\sin\theta$, show that: (5)

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = \left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2$$

(b) If $x = r\sin\theta\cos\varphi$, $y = r\sin\theta\sin\varphi$, $z = r\cos\theta$. Find $\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)}$. (5)

OR

Q3 (a) In a plane triangle ABC find the maximum value of $\cos A \cos B \cos C$. (5)

(b) If $V = f(2x - 3y, 3y - 4z, 4z - 2x)$ Compute the value of $6V_x + 4V_y + 3V_z$. (5)

Q4 (a) Solve the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x x \cos x$. (5)

(b) Find the Orthogonal trajectory of $y^2 = 4a(x + a)$. (5)

OR

Q5 (a) Solve $\frac{d^2y}{dx^2} - y = (1 + \frac{1}{e^x})^{-2}$ by method of variation of parameter. (5)

(b) Prove that $J_0(0) = 1$. (5)

Q6 (a) Investigate the values of a and b so that the equations $2x + 3y + 5z = 9$, $7x + +3y - 2z = 8$, $2x + 3y + az = b$. have: (5)

(i) No solution (ii) A unique Solution (iii) An infinite number of solution

(b) Solve the system of linear equation by Gauss elimination method. (5)

$x + 3y + z = 10$, $x - 2y - z = -6$, $2x + y + z = 10$

$-24 - 2z$

$-24 - 1z$

OR

Q7 (a) Find the eigen value and eigen vector of the matrix A= $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. (5)

(b) Verify Cayley – Hamilton theorem for the matrix:

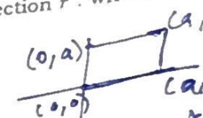
A= $\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}$ (5)

Q8 Verify Stoke's theorem for the function $\vec{F} = x^2\hat{\imath} - xy\hat{\jmath}$. integrated round the square in the plane $z = 0$ And bounded by the lines $x = 0, y = 0, x = a, y = a$. (10)

OR

Q9 (a) Find the arc length of semi cubical parabola $\vec{r}(t) = (t\hat{\imath} + t^{\frac{3}{2}}\hat{\jmath})$ from (0,0,0) to (4,8,0). (5)

(b) Find the directional derivative of $\frac{1}{r}$ in the direction $\vec{r}$. where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. (5)



****

$(c_1 + c_2 x)e$

$x \cdot e^x - e^x$

$\cos x = -\sin x$

$\sin x = -\cos x$

$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial u} \times \frac{\partial u}{\partial x}$

$2 = -6$