

END TERM EXAMINATION

THIRD SEMESTER [B.TECH] DECEMBER-2024

Paper Code: ES-201

Subject: Computational Methods

Time: 3 Hours

Maximum Marks: 60

Note: Attempt five questions in all including Q. no.1 which is compulsory. Select one question from each unit. Scientific calculator is allowed.

- Q1 (a) Determine the decimal number that correspond to the machine word $[45DE4000]_{16}$ (2)
- (b) Using secant method find the root of $x \sin(x) - 3 \cos(x) = 0$ between (0.0, 1.8) with accuracy of 2 digits after decimal point. (3)
- (c) For following given set of data for x & y , formulate the Newton divided differences table. (3)

x	0.00	2.00	4.00	6.0	8.0
y	0.00	8.00	64.00	216.00	512.00

- (d) Compute $\int_0^1 \frac{\sin(x)}{x} dx$ by using the composite Trapezoid rule with six uniform points. Assign the value $\left(\frac{\sin x}{x}\right) = 1$ at $x = 0$. (3)
- (e) Define Decomposition of any matrix. Explain the required condition for Doolittle and Crout decomposition method. (2)
- (f) Find the value of λ for which the system of equations $x+y+4z=1$, $x+2y-2z=1$ & $\lambda x+y+z=1$, will have unique solution. (3)
- (g) Using Euler's method find the value of y at $x=0.10$ of the ODE $\frac{dy}{dx} = x + y + xy$ Where initial conditions are $x_0 = 0.00$ & $y_0 = 1.00$ and using step size 0.10. (3)
- (h) Give an example and conditions of Parabolic, Hyperbolic and Elliptic partial differential equations. (1)

UNIT - I

- Q2 (a) Find the value of function $f(x) = 1 - \cos(x)$ at $x = 0.1$. Modify the expression so that loss of significant digits can be avoided and calculate the value again. Compare two values with the true value 0.4996×10^{-2} . (5)
- (b) Define rate of convergence and stability of iterative method. Prove that the rate of convergence of Newton-Raphson method is 2. (5)
- Q3 (a) Define Multivariate unconstraint minimization problem with an example. Using Newton Method to minimize multivariate function, minimize $f(x^k) = 4x_1^2 + x_2^2 - 2x_1x_2$ starting at $x_0^k = [1, 1]^T$, where $x^k = [x_1, x_2]^T$ (5)
- (b) Determine the minimum point of the function $f(x) = x^2 - 7x + 12$ by Fibonacci search method, if the first uncertainty interval is [2,4] (5)

UNIT-II

- Q4 (a) Explain error estimation in Newton-Gregory Forward interpolation. Also prove that the maximum error in Newton's Forward interpolation is 1 when $|x-x_0| < h$, where h is step size of the given data. (5)
- (b) For the given set of data for X and Y construct the table of forward differences. Find the interpolated value of Y at $X=4.60$ using Newton forward interpolation formula. Also find the estimated error in interpolated value. (5)

X	1.0	2.0	3.0	4.0	5.0	6.0
Y	7.0	13.0	21.0	32.0	48.0	70.0

- Q5 (a) Find $I = \int_0^1 x dx$, by Gaussian Quadrature formula for $n=4$. Where the values of 'abscissae and Weights' corresponding to $n=4$ are given.
Abcissae = $(\pm 0.33998 \text{ \& } \pm 0.86114)$ and corresponding Weights = $(0.65214 \text{ \& } 0.34785)$ (4)
- (b) Evaluate the definite integral $\int_0^1 \frac{1}{1+x} dx$ correct to three decimal point using the basic trapezoidal rule with $h = 0.5, 0.25$ and 0.125 then obtain a better estimate using Romberg's method. Compare the results with the true value. (6)

UNIT-III

- Q6 (a) Test the consistency of the system of following given equations:

$$\begin{cases} 5x_1 + 3x_2 + 7x_3 = 4 \\ 3x_1 + 26x_2 - 2x_3 = 9 \\ 7x_1 + 2x_2 + 10x_3 = 3 \end{cases}$$

If the system is consistent, is the system have finite solution or infinite solution? (4)

- (b) Explain the concept of partial pivoting and hence solve the following given system of linear equation by Gauss-Jordan method. The system of linear. (6)

$$\begin{aligned} &x_1 + x_2 + x_3 = 1 \\ \text{equations is: } &4x_1 + 3x_2 - x_3 = 6 \\ &3x_1 + 5x_2 + 3x_3 = 4 \end{aligned}$$

- (c) Obtain the linear Spline for the function $f(x)$ define by the data given below

x	1	2	4	8
f(x)	3	7	21	73

Evaluate the function at $x=3$. (4)

- (d) Solve the following given system of linear equations using Cholesky factorization method: (6)

$$\begin{aligned} 25x + 15y - 5z &= 35 \\ 15x + 18y &= 33 \\ -5x + 11z &= 6 \end{aligned}$$

UNIT-IV

Q8 (a) Using Picard method, compute $y(0.2)$ to three decimal precision from $\frac{dy}{dx} = x + y$ given that $y(0) = 1$. Compare the result with the exact result for the value of y at 0.2. (4)

(b) Consider the initial value problem $\begin{cases} \frac{dx}{dt} = t + x^2 \\ x(0) = 1 \end{cases}$ Using Runge-Kutta method of 4th order find $x(0.2)$ taking $h=0.1$. (6)

Q9 (a) Solve the initial value problem $\frac{dy}{dx} = x - y^2$ in the range $0 \leq x \leq 1$, for the initial condition $y(0) = 0$. (4)

(b) Solve the elliptical equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values given as (6)


