

# END TERM EXAMINATION

THIRD SEMESTER [B.TECH] DECEMBER 2025 - JANUARY 2026

Paper Code: ES-201

Subject: Computational Methods

Time: 3 Hours

Maximum Marks: 60

Note: Attempt five questions in all including Q. No.1 which is compulsory. Select one question from each unit. Scientific calculator is allowed.

Q1 Attempt **any four** of the following questions:

2.2 (a) Explain the rapid convergence of Newton-Raphson iteration by showing that the convergence is "quadratic". (5)

1.1 (b) Use the Newton methods to Maximize:  $z = -(x_1 - \sqrt{5})^2 - (x_2 - \pi)^2 - 10$  to within a tolerance of 0.05. (5)

(c) Compute the integral  $I = \int_{-2}^2 e^{-x^2} dx$  using 3-point Gaussian quadrature. (5)

2. (d) Using the data  $\sin(0.1) = 0.09983$  and  $\sin(0.2) = 0.19867$ , find an approximate value of  $\sin(0.15)$  by Lagrange interpolation. Obtain the maximum truncation error bound. (3+2)

3. (e) Estimate the computational efforts required in terms of arithmetic operations in Gauss elimination method to solve system of linear equations during elimination process and substitution process and solve  $3x_1 + 2x_2 = 18$   
 $-x_1 + 2x_2 = 2$  using Gauss elimination method. (3+2)

(f) State whether the following piecewise polynomial is spline or not.  $f(x) = \begin{cases} x^2 + 1 & 0 \leq x \leq 1 \\ 2x^2 & 1 \leq x \leq 2 \\ 5x - 2 & 2 \leq x \leq 3 \end{cases}$  (5)

2.1 (g) Estimate the value of  $y(0.20)$  by Picard's method, given that  $\frac{dy}{dx} = x + y + xy$  and  $y(0.0) = 1.00$  taking  $h = 0.05$ . (5)

(h) Using Modified Euler's method, find  $\dot{y}(0.2)$  correct to four decimal points given that  $y' = y + e^x, y(0) = 0$ . (5)

## UNIT-I

2 (a) Find the value of function  $f(x) = 1 - \cos(x)$  at  $x = 0.1$ . Modify the expression so that loss of significant digits can be avoided and calculate the value again. Compare two values with the true value  $0.4996 \times 10^{-2}$ . (6)

2 (b) Explain Lagrange's mean values theorem and verify the statement for the function  $f(x) = \frac{1}{3}x^3 + 2x$  in  $[0,3]$ . (1.5+2.5)

Q3 Estimate the minimum value of  $f(x,y) = x^3 - 4x + y^2 - xy$  using Nelder-Mead method up to five iterations. Given initial vertices are  $v_1 = (0,0), v_2 = (1.2,0)$  and  $v_3 = (0,0.8)$ . (10)

## UNIT-II

Q4 (a) Prove that  $\Delta^n \sin(ax + b) = \left(2 \sin\left(\frac{ah}{2}\right)\right)^n \sin\left[ax + b + n\left(\frac{\pi+ah}{2}\right)\right]$ ,  $h$  being the interval of differencing. (6.5)

(b) For given  $f(x) = \frac{1}{x^2}$ , find the divided differences  $[a,b]$  and  $[a,b,c]$ . (3.5)

- Q5 (a) If the integral  $\int_0^{\pi} e^{\cos x} dx$  is to be computed with absolute error less than  $\frac{1}{2} \times 10^{-3}$ , by composite trapezoidal rule with a uniform partition, how many subinterval are required to compute the value less than  $\frac{1}{2} \times 10^{-3}$ . (6)
- (b) Use Romberg's method to estimate  $\int_0^1 \frac{1}{1+x} dx$  correct to three decimal taking  $I(0.5) = 0.7084$ ,  $I(0.25) = 0.6970$  and  $I(0.125) = 0.6941$ . Compare the results with the true value. (3+1)

## UNIT-III

- Q6 (a) Test the consistency of the system of following given equations:
- $$\begin{cases} x_1 + 4x_2 + \alpha x_3 = 6 \\ 2x_1 - x_2 + 2\alpha x_3 = 3 \\ \alpha x_1 + 3x_2 + x_3 = 5 \end{cases}$$
- Find the value of  $\alpha$  so that system has unique solution, no solution and infinitely many solution? (5)
- (b) Determined the largest eigen value and corresponding eigen vector of the matrix  $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$  using power series method taking starting eigen vector  $X^0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ . (5)

- Q7 (a) Obtain the linear Spline for the function  $f(x)$  define by the data given below:

$x$	1	2	4	8
$f(x)$	3	7	21	73

Evaluate the function at  $x=3$ . (5+1)

- (b) Determine the Cholesky factorization of the matrix A, where A is:
- $$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix}$$
- (6)

## UNIT-IV

- Q8 (a) Using the Taylor series for  $y(x)$ , compute  $y(0.1)$  correct to four decimal places if  $y(x)$  satisfies  $y' = x - y^2$  and  $y(0) = 1$ . Compare the result with the exact result value of  $y(0.1)$ . (4+1)

- (b) Consider the initial value problem  $\begin{cases} \frac{dx}{dt} = t + x^2 \\ \text{where } x(0) = 1. \end{cases}$  Using Runge-Kutta method of 4<sup>th</sup> order find  $x(0.2)$ . 1.2848 (5)

- Q9 Apply Milne's method, to find the solution of the first order differential equation  $\frac{dy}{dx} = x - y^2$  in the range  $0 \leq x \leq 1$  for the boundary conditions  $y=0$  at  $x=0$  taking  $h=0.2$ . Find first three required values  $y_1, y_2, y_3$  and corresponding  $f_1, f_2$  &  $f_3$  by Picard method. (10)