Exam Roll No. ....

(Please write your Exam Roll No.)

19/12/25

END TERM EXAMINATION

THIRD SEMESTER [B.TECH] DECEMBER 2025-JANUARY 2026

Subject: Discrete Mathematics

Paper Code: CIC-205 **Maximum Marks:60**

Time: 3 Hours

Note: Attempt all questions as directed. Internal choice is indicated.

Q1 Attempt **any five** of the following questions: **(4x5=20)**

3 a) Let A= {1,2,3}and R= {(1,1), (2,2), (3,3), (1,2)}
   i) Is R reflexive?
   ii) Is R symmetric?
   iii) Is R transitive?

2 b) What is the principle of inclusion-exclusion? Provide the formula for two sets.

c) Define generating functions and give an example. List two applications of divide-and-conquer algorithms.

3 d) What is the Master's theorem? State its importance in algorithm analysis.

e) Prove that the intersection of two subgroups of a group G is also a subgroup of G.

f) Explain Lagrange's theorem for finite groups.

2 g) Prove that for any connected planar graph, V−E+F=2, where V is the number of vertices, E the number of edges, and F the number of faces.

3 h) What is a spanning tree? Give an example of a connected graph with 5 vertices and find its minimum spanning tree (MST).

Q2 Using Pigeon hole Principle Prove that in any group of 13 people, at least two people were born in the same month. Also Prove that $\sqrt{2}$ is irrational. **(10)**

OR

Q3 Define an equivalence relation R on the set Z as: **(10)**
a ~b if a−b is divisible by 3.
a) Verify that R is an equivalence relation.
b) Find the equivalence classes of 0, 1, and 2.
Convert the following formula to Conjunctive Normal Form (CNF):
$(P \rightarrow Q) \wedge (\neg R \rightarrow P)$

Q4 Define a lattice. Show that the set {2,4,8,16} with divisibility as the partial order forms a lattice. Minimize the following Boolean expression using Karnaugh map: $F(A,B,C)=\Sigma(0,1,2,5,7)$ **(10)**

OR

Q5 Analyze the time complexity of Merge Sort using recurrence relations and solve it find the time complexity of the following recurrence relation: $T(n)=4T(n/2)+n^2$ **(10)**

Q6 Define a group homomorphism. Prove that the image of a group homomorphism is a subgroup of the codomain. In a group GGG, prove that the inverse of each element is unique. **(10)**

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OR

Q7 Let G= {e,a,b} be a group of order 3 under multiplication. Use Cayley's theorem to show that G is isomorphic to a subgroup of the symmetric group S_3. **(10)**

Q8 Analyze and compare the time complexities of Prim's algorithm and Kruskal's algorithm. Which one would you prefer for dense graphs? Why? Also Prove the Five Color Theorem for planar graphs. **(10)**

OR

Q9 Write an algorithm to find the connected components of an undirected graph. Apply your algorithm to the following graph: **(10)**
V={1,2,3,4,5,6},
E={(1,2),(2,3),(4,5)}.
Does the following graph contain a Hamiltonian circuit? Justify your answer:
V={1,2,3,4},
E={(1,2),(2,3),(3,4),(4,1),(1,3)}.

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